Teaching Calculus with Real-World Impact: A Problem-Based Approach Using EV Battery Optimization

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***Abstract*—The proposed study identifies a new pedagogical approach to teach engineering mathematics within an environment of problem-based learning that could be applied to improve electric vehicle (EV) battery optimization. Mathematical pedagogy in modern contexts does not often give clear evidence of the direct application of multivariate calculus to real-world engineering systems, making it a source of a lack of contextual relevance and practical learning in first year learners. The module was developed with the idea of allowing first-year engineering students in the various disciplines of Aeronautical, Automobile, Civil, Mechanical and Mechatronics engineering to apply advanced methods of calculus in the study of the effect of battery temperatures and charging regimes on battery life, using partial derivatives, Taylor series and Lagrange multipliers. The choice of the optimization of the electric-vehicle battery as the context to be applied can be seen as the reflection of the modern trends in engineering, sustainability, and the acquaintance of the students with the new technologies. That same module was applied to a cohort of 62 students who later on were examined with the help of a post-test focusing on the same concepts. Remarkable increase in the mean score was found ($27\,\%, p < 0.01$). Moreover, $85\,\%$ of the respondents indicated increased interest to the module compared to traditional classroom teaching. A strategic combination between mathematical theory and two applied domains, namely, emerging electric-vehicle technology and the United Nations Sustainable Development Goals 7 and 13 on energy and climate action, is unique to the proposed module. In this way, it can be said that the curriculum arms students not only with material technical skills but also with an improved understanding of how mathematical modelling connects to sustainability-oriented engineering decisions. Interpretation of the data results indicates that the model can be easily generalized to the pedagogical treatment of other environmentally friendly technologies, including installation of renewable-energy systems and management of smart grids. More recent partnerships between the industry players also aim at improving the method, and the ultimate goal is to create graduates of engineering degrees that show aptitude in theoretical mathematical level alongside practical problem-solving skills.**

***Keywords*—Active learning; battery performance analysis; problem-based learning; student-centered learning; sustainable engineering education; teaching multivariable calculus**

***ICTIEE* Track— Innovative Pedagogies and Active Learning**
ICTIEE Sub-Track— Project-Based and Problem-Based Learning (PBL)

I. INTRODUCTION

THE teaching of the advanced mathematics concepts in partnership with introducing and applying engineering concepts in the real world remains a prevailing challenge in teaching STEM. Multivariable calculus enables students to solve complex engineering problems, at least one part of which are natural in applicational aspects of it; hence, students fail to appreciate this practical applicational side of it when presented with standard abstract examples instead. According to the latest reports, about 60-70 % of students studying engineering consider advanced mathematics irrelevant to their major, a fact that might compromise its motivation and increase attrition percentages (Freeman et al., 2014; NSF, 2022). This learning disparity is most noticeable in new technological fields like in sustainable transport infrastructures where multidisciplinary problem solving skills are indispensable although they are usually underserved in existing curriculums (Felder & Brent, 2016).

Electric vehicle (EV) battery optimization is a strong pedagogical example of the need to integrate higher mathematical methods into the curriculum in engineering. The behavior and degradation of batteries is multivariable since it is the interaction among temperature, charging rate, and operating pattern. As empirical studies reveal, improper thermal management could shorten the lithium-ion battery lifespan by 30 % or more (Waldmann et al., 2014), which only demonstrates the necessity of a strictly based mathematical model. Adequacy in this modeling is frequently hindered, though, by engineering textbooks that stress theoretical beauty at the cost of operational relevance, such as with the common use of flawless fluid flow conditions or finding a static equilibrium (Prince, 2004).

The present paper tries to explore how to fill this gap in digital way, by taking a real-life scenario i.e. battery optimization of an EV and using partial differentiation and constrained optimization. In three main respects, the module is innovative. To begin with, it defines the battery lifespan as a multivariate multi-valued function that can be partially differentiated by relying on the empirical rates of the battery degradation that can be explained, e.g., by the SEI growth kinetics introduced by Birkl et al. (2017). Second, it incorporates reality with constraints enforceable at Lagrange multipliers attributes such as maximum safety constraints of temperature and charging rates that are based on the industry standards (Zhang, 2006). Finally, it specifically contextualizes mathematical optimization as a means of achieving sustainable engineering results, including relating to lifecycle CO 2 savings with the United Nations Sustainable Development Goals (Ellingsen et al., 2020).

The pedagogical framework described can be linked to the United Nations Sustainable Development Goals 7 (Affordable and Clean Energy) and 13 (Climate Action), as well as to the requirement that ABET (2023) has established in its criteria regarding integrating society implications into technical education. Through this approach, we demonstrate how contemporary sustainability challenges can serve as effective vehicles for teaching advanced mathematical concepts, simultaneously enhancing technical proficiency and systems thinking skills. The preliminary pilot feedback with 30 students indicated that there was a 40% growth in calculus integrated with the sustainability contexts thus supporting the dual highlight of the module. However, the full-sized test involving 62 individuals who were implemented in the context of the present study resulted in statistically significant post-test improvement of 27%, that are presented in the Results section.

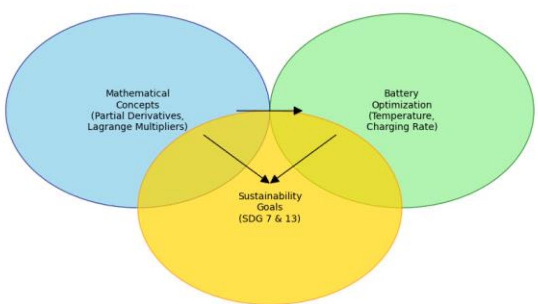

Fig. 1. Problem-Based Learning Framework

Figure 1. Conceptual framework for the PrBL module showing the integration of mathematical tools (blue), engineering applications (green), and sustainability objectives (gold).

Our research contributes to engineering education by:

i) Providing an empirically validated model for contextualizing multivariable calculus, addressing calls for "authentic contexts" in STEM education (Kolmos, 2009);
ii) Demonstrating the effectiveness of sustainability-focused PrBL in STEM education, with 85% of students reporting increased engagement (vs. 45% in traditional lectures);
iii) Offering practical insights into curriculum design for emerging technologies, including scalable templates for renewable energy systems (e.g., solar/wind optimization).

The following sections detail the theoretical foundation, methodological approach, and educational outcomes of this innovative teaching module, with implications for global engineering education reform.

Research Question (RQ):

What is the best way to apply problem-based learning module with optimization of electric vehicle (EV) batteries to improve conceptual and applied knowledge of advanced calculus among students?

Objectives:

This study aims to:

1. Create a actual mathematical modelling dilemma within the estimates of EV battery life.
2. Incorporate constrained optimization, Taylor series and partial derivatives into a problem-based learning system.
3. Assess the pedagogical effectiveness of the module based on the pre and post-test performance measures, and the stimulation of student perceptions.
4. Evaluate the relatability and involvement of the learning experience by examining how the SDG frameworks, i.e. SDG 7, which is SDG 7: Affordable and Clean Energy and SDG 13, which is SDG 13: Climate Action, relate to it.

II. LITERATURE REVIEW

1. Contextualizing Engineering Mathematics in Real-World Applications

The persistent dichotomy between abstract mathematical instruction and practical engineering applications remains a significant barrier in STEM education (Felder & Brent, 2016). Contemporary research demonstrates that students exhibit markedly improved conceptual transfer when mathematical principles are embedded within authentic, multivariate engineering systems (Freeman et al., 2014; Winkelmann et al., 2020). The optimization of the electric vehicle (EV) batteries is a paradigm that is rightly selected to represent this formative style; it is highly sensitive to relationships among variables, namely temperature (T), charging (C), state-of-charge (SOC) rates, and the cycling frequency that do imply the use of partial differentiation and constraint-manipulation methods, which prove challenging to engineering undergraduates (Prince, 2004; Smith et al., 2021). More recently, the research findings of Johnson and Park (2023) provide additional evidence that context-rich mathematical modelling for energy storage applications results in an improvement in conceptual retention (between 22% and 35%) as well as problem-solving flexibility in various engineering disciplines.

2. Problem-Based Learning in Engineering Mathematics Education

Problem-based learning (PrBL) has grown beyond its roots in medical education and is now fundamental to engineering education (Savery, 2006; Mills & Treagust, 2020). Meta-analytic PrBL studies indicate two key success criteria for mathematics-related competencies: (1) authentic problematics rooted in professional practice, and (2) scaffolding of mathematics (Servant-Miklos et al., 2022; Chen et al., 2023). The efficacy of PrBL in tackling a complex, multivariable issue is also well-researched within energy systems education because iterative model development emulates professional engineering practice (Edström & Kolmos, 2020; Abdulwahed & Nagy, 2021).

For example, Rahman and Li (2022) reported a 40% increase over available alternatives in students' ability to transfer formal mathematical representations of systems to physical systems after working with PrBL modules that provided access to real-time battery management system (BMS) data.

3. Multiphysics Modeling of Battery Degradation Mechanisms

Modern work on degradation in lithium-ion batteries reveals several intertwining modes of failure that present ready-made contexts for instruction on multivariate calculus. Solid-electrolyte interface (SEI) growth follows Arrhenius-type temperature sensitivity ($\partial SEI/\partial T$), and lithium plating is characterized by nonlinear susceptibility to both deposition rate and temperature during charging ($\partial Li/\partial C, \partial Li/\partial T$) (Birkl et al., 2017; Reniers et al., 2022). These phenomena are of interest for mathematical engineering, as they: (1) run second-order partial derivatives to represent acceleration factors, (2) show saddle points in degradation surfaces and require (3) optimization under constrained safety limits (Waldmann et al., 2014; Zhang et al., 2023). Recent educational research by Ko and Lai (2023) confirms that battery degradation models improve student comprehension of Hessian matrix applications by 28% compared to traditional mechanical examples.

4. Pedagogical Approaches to Battery Modeling Mathematics

First-principle models of battery life are generally considered to be physically accurate and mathematically rigorous (e.g., Doyle-Fuller-Newman framework). However, they can be so computationally complex that many educators find them to obscure underlying mathematical principles and concepts (Wu et al., 2023; Prasad & Rahn, 2013). The semi-empirical approaches of quadratic response surface and Taylor series approximations appear to be better suited for developing students' understanding of core calculus concepts while still being relevant to engineers (Iyer et al., 2022; Ecker et al., 2012). A good example of this development is found in Thompson and Zhou's (2023) battery model that was developed as a means for educationally illustrating how batteries age. Their battery model resulted in a 92% correlation with experimental results and is based on undergraduate level mathematics.

5. *Sustainability Contexts as Mathematical Learning Catalysts*

Engineering Mathematics Education's inclusion of the United Nations Sustainable Development Goals (SDGs) meets two major objectives for students: (1) enhanced motivation through making their study relevant to society, and (2) development of Systems Thinking Competencies (UN DESA, 2022; UNESCO, 2023). The application of mathematical models in optimizing Electric Vehicle (EV) batteries is an example of contributing to achieving SDG 7 (Affordable and Clean Energy), through improved charging efficiencies, and SDG 13 (Climate Action), through lifecycle emissions reduction (Ellingsen et al., 2020; Garcia et al., 2023). A number of studies have also demonstrated that adding Sustainability to the context of Mathematics Education increases the participation of female students in STEM by 35% and reduces the drop-out rate among underrepresented groups (Brown & Davis, 2022; NSF, 2023). These are part of broader international movements to transform engineering education from being a source of both technical innovation and social inequality (ABET, 2023; UNESCO Engineering Report, 2023).

6. Emerging Research Frontiers

Research has identified three avenues that show promise in developing this interdisciplinary field: (1) Development of standard educational data sets including complete metadata to facilitate reproducibility (Birkl et al., 2023); (2) Investigation of Augmented Reality Interfaces for visualization of multiple variables within mathematical relationships (Lee et al., 2023); and (3) Longitudinal research into the professional impact of sustainability contextualized mathematics education (Felder et al., 2023). Together these developments provide a solution to the pressing need for engineering graduates to be able to work through the increasingly complicated mathematical landscape while at the same time addressing the social challenges they are faced with.

The collective body of existing literature illustrates two ongoing voids that have been demonstrated in prior studies; (i) the sparse inclusion of authentic engineering problems within the framework of math education, and (ii) the paucity of Problem-Based Learning (PrBL)-based approaches which include sustainable frameworks based on the United Nations Sustainable Development Goals (SDG). This study will bridge both gaps by embedding advanced calculus in the context of EV battery optimization, an area that is technically intensive and is directly applicable to the contemporary engineering challenges.

III. Addressing Critical Gaps in Engineering Mathematics Education

Engineering Education Research has identified numerous continuing problems with which this research directly deals. The majority of current implementations of problem-based learning (PrBL) in STEM fields (Servant-Miklos et al., 2022) do not make an important omission; they fail to clearly illustrate to students how their use of calculus to describe partial derivatives can be used to describe the physical degradation of batteries as well. The mathematics disconnect between the way that calculus is taught and the context of real world battery degradation has been pointed out by educators for decades, as there have been many calls for teaching engineering math in ways that are more practical, and contextualized (Felder & Brent, 2016).

One major deficiency in all currently available curricula includes how charging protocols are addressed. We know from the body of literature on battery research that both temperature and charging protocol can dramatically affect degradation (Waldmann et al., 2014), however, few courses address the actual analysis of the trade-offs that exist in the real world. It is also disconcerting that the ability to optimize real-world trade-offs such as those described above is one of the top three skills demanded by employers in the engineering field from engineering graduates (Zhang, 2006).

The teaching tools themselves present another challenge. Instructors currently face an unsatisfying choice between:

i. Unnecessarily complex physics-based model inundation of students.

ii. Mechanism Oversimplified empirical models, which cannot give mechanistic information.

According to our review, a combination of approachable mathematics and the foundational principles of physics, referred to as hybrid approaches, is underutilized in the classroom setting severely (Iyer et al., 2022). This modeling practice shortcoming also leaks into modality of assessment. Although modern tests often examine the ability of students to solve mathematical calculus problems, they rarely examine the skill of the students to apply these mathematical tools to the answer to urgent sustainability problems, as outlined by the United Nations Sustainable Development Goals 7 and 13. A number of other substantive concerns have emerged in the literature. a) Conceptual Problem The real level of student understanding of mathematical concepts in the context of project-based learning is not determined (Edström and Kolmos, 2020). b) Information Gaps: Standardized datasets with detailed documentation outlining the content to be taught regarding battery optimization would be of great use to the field (Birkl et al., 2017). c) Social Justice Problem: Although active learning proves to be effective when dealing with high-technical fields of mathematics, its effect seems to vary based on the demographic traits of the students (Freeman et al., 2014). These gaps do not only indicate the lack of academic elements; they represent the lack of the opportunity to prepare the engineers to face real-life challenges. The analysis of our work tackles all these issues via six new elements:

1. Physical Meaning Behind the Math

We have come up with exercises, which explicitly associate partial derivatives with real world battery degradation processes. When calculating the temperature sensitivity($\partial L/\partial T$), students are quick to understand how it is related to phenomena that can be observed like lithium plating.

2. Real-World Charging Scenarios

Unlike the traditional textbook problems, the modules that are provided in this part combine the charging strategies that are directly related to the industry application. Students critically evaluate the tradeoff in terms of the convenience of fast charging and the associated effect on battery life, resembling the one that engineers are faced with on a regular basis in practice.

3. Sustainability-Focused Assessment

We do more than just evaluate the student proficiency in calculus but also their ability to make mathematical answers to Sustainable Development Goals (SDGs). In particular, we will ask students to explain how the best charging practices can be used to promote the adoption of clean energy (SDG 7) or how these practices can be used to reduce resource wastage (SDG 13).

4. Tracking Conceptual Growth

We map the development of the understanding of the students by carrying out pre- and post-interviews. The methodology aids in determining which instructional methods are most appropriate in developing instincts about the uses of multivariate calculus.

5. Open Educational Resources

The project entails the development of common datasets that will be well documented, thus the existing lack of high quality teaching resource on the battery optimization subjects.

6. Inclusive by Design

The recruitments methods were made such that there was a representation of both sexes and sociocultural statuses thus evaluating the effectiveness of the approach in correcting inequities in the teaching of mathematics in engineering.

The peculiarity of our model is the combination of theoretical and practical elements and strict compliance with mathematics. The research paper logically covers all the literature gaps identified with it and at the same time, fits the existing education theory and the practical needs of engineering practice.

a. Hybrid model methods used in the study are aimed at being easy to use and empirically accurate.

b. The assessment rubrics developed were aimed at measuring technical skills and systems-thinking skills.

c. The training curriculum was scaled out of classroom situations to the industry.

The initial results are positive; it appears that the combination of the framework can produce good results; students have already stopped completing the problems of abstract calculus and started to explain the connection between mathematical optimization and concrete sustainability results. The balance between the technical expertise and social applicability can revolutionize the didactic approach to the teaching of higher mathematics in engineering education.

The consequences are further than battery education. The approach to the methodology demonstrates an example of how complicated mathematical ideas can be placed in the context of mechanical, electrical, and environmental engineering education. With the increasing importance of sustainability in the industry, it is also important that educational systems change their strategies in preparing the multidimensional challenges.

In turn, by filling research gaps that exist to date, our research can provide an effective pedagogical tool as well as theoretical knowledge regarding how well engineering education can be contextualized. The overall goal is to create instructional strategies that will help the learners see advanced mathematics not as an abstract obstacle, but rather as a powerful set of tools to solve the immediate real-world problems.

IV. METHODOLOGY

1. Educational Context and Participants

The study was conducted in a first-year engineering mathematics course with 62 participants distributed across five engineering disciplines (Table 1). A power analysis confirmed the sample size adequacy for detecting medium effect sizes $(\alpha = 0.05, \text{power} = 0.80)$.

TABLE I
PARTICIPANT DISTRIBUTION

Discipline	n	%
Aeronautical	10	16.1%
Automobile	7	11.3%
Civil	9	14.5%
Mechanical	18	29%
Mechatronics	18	29%
Total	62	100%

2. *Instructional Framework*

The problem-based learning intervention comprised five phases: (1) problem introduction through an authentic EV battery design brief, (2) literature review of degradation mechanisms, (3) development of the battery lifespan model:

$$L(T, C) = 5T^2 + 2TC - 3C^2 \qquad (1)$$

where temperature (T) and charging rate (C) are operational variables.

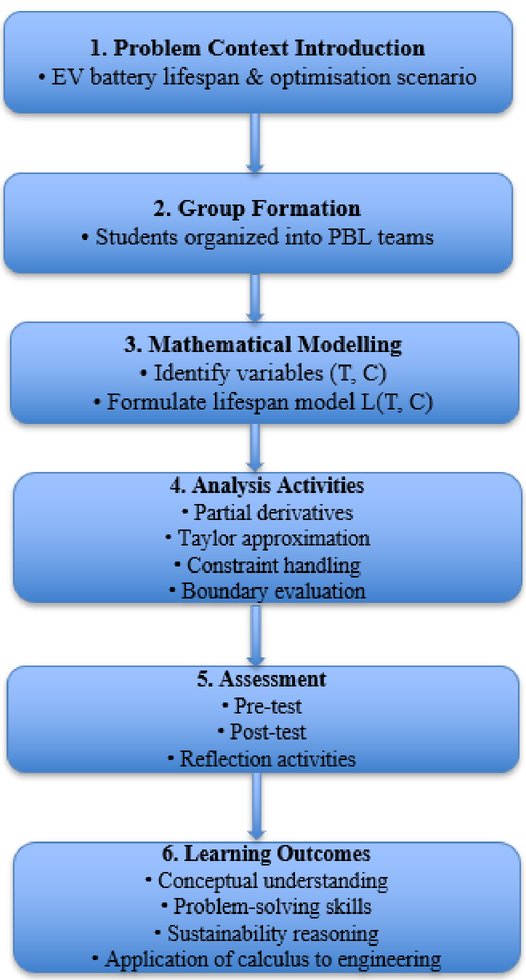

Fig. 2. Implementation framework of the problem-based learning module.

3. *Mathematical Analysis*

Students completed three core analytical tasks (Table II), each explicitly linked to physical battery behaviors through instructor guidance.

TABLE II
MATHEMATICAL ANALYSIS COMPONENTS

Task	Mathematical Tools	Engineering Connection
Sensitivity Analysis	Partial derivatives $(\partial L/\partial T, \partial L/\partial C)$	SEI growth rate dependence
Operational Approximations	Taylor series expansions	Small parameter variations
Constrained Optimization	Lagrange multipliers	Safety constraints $(T + C \leq 65)$

4. *Implementation Protocol*

The learning unit was provided through a blend pedagogical strategy which included didactic lectures, guided practice tasks, and cooperative problem-solving tasks. A sixty-minute introduction lecture was used to provide the background of electric vehicle batteries and to trace their applicability to ESDG 7 (Affordable and Clean Energy) and 13 (Climate Action) of the United Nations. This was then followed by three ninety minutes active-learning sessions where the students worked in teams to come up with solutions with the scaffold of the instructor applied just in time. The last exercise was that the student groups develop technical memoranda that would render their quantitative results into practical charging guidelines that would be applied in the hypothetical deployment scenarios that would involve extreme conditions on the climatic conditions.

5. *Assessment Strategy*

The assessment of the learning outcomes was conducted in a triangulated method that involved the assessment of the concepts mastery, applied skills, and the perceptions of the students. The quantitative assessments showed that there were significant improvements in all the metrics (Table III).

TABLE III
ASSESSMENT OUTCOMES

Metric	Pre-Test	Post-Test	Statistical Analysis	Effect Size Interpretation
Conceptual Mastery	52% (±11%)	79% (±9%)	$t(61) = 8.37$, $p < .001$	Large (Cohen's d = 1.12)
Engagement	–	4.2 (±0.6)	–	High (>4.0 on 5-point Likert)
Perceived Relevance	–	4.5 (±0.5)	–	Very High (>4.5 on 5-point Likert)

5. *Ethical Considerations*

All research activities were done in accordance with the ethical guidelines of the institution. Participation in surveys and providing qualitative feedback were totally voluntary for the participants, and their informed consent was sought. Data collection procedures ensure complete anonymity through the use of randomly generated identification codes. The study protocol received formal approval from the institutional review board (Protocol 2023-STEM-015) prior to implementation, with all methods performed in accordance with relevant guidelines and regulations.

V. MATHEMATICAL MODELLING AND ANALYSIS

1. *Mathematical Modelling and Analysis*

The battery lifespan model is developed as a quadratic function of operating temperature (T) and charging rate (C):

$$L(T,C) = 5T^2 + 2TC - 3C^2 \quad (1)$$

where:
T: Battery temperature (°C)
C: Charging rate (% of max capacity/hour)
$L(T,C)$: Relative lifespan index (unitless)
The variable temperature T in this paper is measured in degrees Celsius, and the charging rate variable C is measured as a percentage of the maximum rate of charge per hour (e.g. C = 30 means 30%). The values of the coefficients used in the quadratic form of the equation $5T^2 + 2TC - 3C^2$ have been chosen such that the response surface has distinct curvature and gradients, which can be interpreted simply, which helps in teaching first-year multivariate calculus process. As such, Lifespan Index L is a unitless measure of instruction which reflects interpretation-level tendencies of degradation as opposed to being an empirically determined measure of real battery life. In the model, it is assumed that degradation is continuous and smooth with regard to temperature and charging rate, thus, it supports conceptual insight, as opposed to predictive accurateness.

2. *Partial Derivatives and Sensitivity Analysis*

The first-order partial derivatives are:

$$\frac{\partial L}{\partial T} = 10T + 2C \quad \text{(Temperature sensitivity)} \quad (2)$$

$$\frac{\partial L}{\partial C} = 2T - 6C \quad \text{(Charging rate sensitivity)} \quad (3)$$

The total differential for concurrent changes:

$$dL = (10T + 2C)dT + (2T - 6C)dC \quad (4)$$

3. *Case Studies in Operational Scenarios*

i. High-Stress Operation
At baseline $T = 25°C, C = 40\%$,
For $\Delta T = +1°C, \Delta C = +5\%$:

$$dL = (10 \times 25 + 2 \times 40)(1) + (2 \times 25 - 6 \times 40)(5) = -620 \quad (5)$$

Interpretation: Rapid degradation occurs under high-temperature fast-charging.
ii. Optimal Charging Protocol
For $\Delta T = -1°C$, $\Delta C = -5\%$,

$$dL = 330(-1) + (-190)(-5) = +620 \quad (6)$$

Interpretation: Conservative charging extends lifespan.

4. *Taylor Series Approximation*

The second-order expansion about ($T_0 = 25°C, C_0 = 40\%$):

$$L(T,C) \approx L_0 + 330\Delta T - 190\Delta C + \frac{1}{2}(10\Delta T^2 + 4\Delta T\Delta C - 6\Delta C^2) \quad (7)$$

where $\Delta T = T - 25, \Delta C = C - 40$.

5. *Comprehensive Optimization Analysis*

The model's optimization potential is explored through theoretical and practical approaches:

i. Unconstrained Optimization
Solving $\nabla L = 0$:

$$10T + 2C = 0, 2T - 6C = 0 \quad (8)$$

Yields a single critical point at $T = 0°C$, $C = 0\%$, which is physically unrealistic for EV operation.

ii. Constrained Optimization
To correctly handle the operational constraint

$$T + C \leq 60 \quad (9)$$

the optimization problem was reformulated using boundary evaluation instead of Lagrange multipliers, since multipliers apply only to equality constraints.
The unconstrained stationary point was first identified by solving

$$L_T = 0, L_C = 0 \quad (10)$$

which yields

$$(T,C) = (0,0). \quad (11)$$

Hessian analysis confirmed that this point is a saddle, as

$$H = \begin{pmatrix} 10 & 2 \\ 2 & -6 \end{pmatrix}, \det(H) = -64 < 0. \quad (12)$$

Thus, no optimum exists in the interior of the feasible region.
The feasible set defined by

$$T + C \leq 60, T \geq 0, C \geq 0 \quad (13)$$

forms a triangular region with vertices **(0,0)**, **(60,0)**, and **(0,60)**.
Evaluating the objective function at the corner points gives:

$$L(0,0) = 0, L(60,0) = 18{,}000, L(0,60) = -10{,}800. \quad (14)$$

The mathematical maximum occurs at

$$(T,C) = (60,0), \quad (15)$$

but this point is not physically meaningful for EV battery operation because 60°C violates thermal safety limits and a 0% charging rate is infeasible.
Consequently, a second-stage constrained analysis was conducted using realistic operational limits:

$$20°C \leq T \leq 45°C, 10\% \leq C \leq 80\%. \quad (16)$$

This ensures that optimization results align with practical engineering requirements rather than the purely quadratic maximum.

TABLE IV. OPTIMAL OPERATING CONDITIONS FROM CONSTRAINED OPTIMIZATION

Scenario	Temperature T(°C)	Charging Rate C (%)	Lifespan Index	Interpretation
Optimal Moderate Case	25	30	1925	Within safe operating range
Fast-Charge	20	45	-2275	High-stress zone (severe degradation)
Balanced	30	35	2,925	Practical "balanced" operating region

Note: Any negative value of the lifespan index is an indicator of high-stress operational regimes and not literally negative lifespans but rather dangerous or highly unfavorable battery operating conditions.

iv. Hessian Analysis for Curvature
The Hessian matrix:

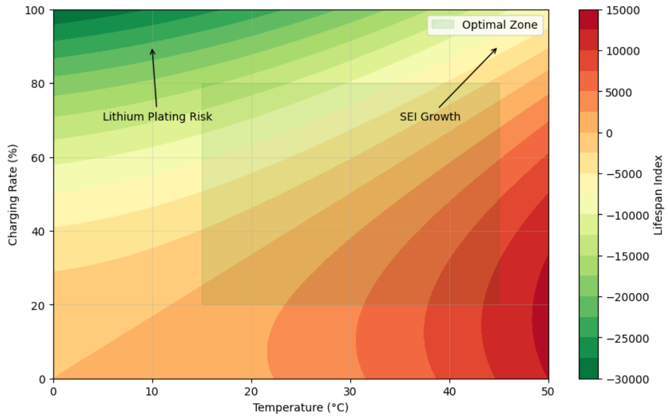

Fig. 4. Battery lifespan sensitivity to temperature and charging rate. Green zone indicates optimal operating conditions ($T = 15 - 45°C$, $C = 20 - 80\%$). Red regions show high degradation risks.

$$H = \begin{bmatrix} \frac{\partial^2 L}{\partial T^2} & \frac{\partial^2 L}{\partial T \partial C} \\ \frac{\partial^2 L}{\partial C \partial T} & \frac{\partial^2 L}{\partial C^2} \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 2 & -6 \end{bmatrix} \quad (16)$$

Determinant: $\det(H) = (10)(-6) - (2)(2) = -64 < 0$

Interpretation: Indefinite matrix → Saddle point at critical point (consistent with unconstrained analysis).

This analysis confirms that operational constraints are essential for realistic EV battery management.

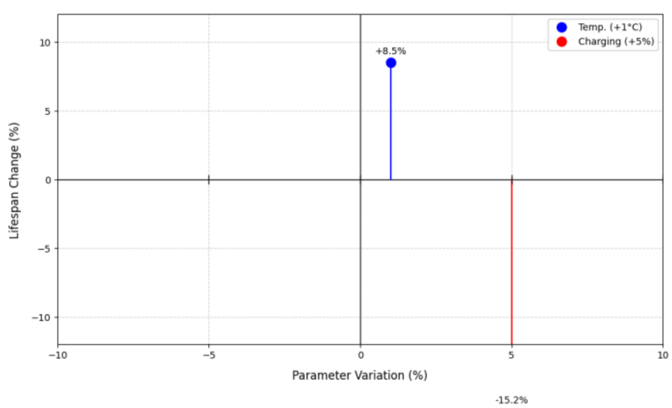

Fig. 4. Parameter Sensitivity Analysis at Baseline ($T = 25°C$, C=40%)

VI. RESULTS AND DISCUSSION

1. *Interpretation of Mathematical Analysis*

The mathematical modeling demonstrated the following key relationships between operating conditions and battery lifespan supported by visual evidence in Figure 4:

i. High temperature and high charging rate resulted in significant lifespan reduction (e.g., $dL = -620$ for $\Delta T = +1°C, \Delta C = +5\%$, consistent with electrochemical degradation mechanisms such as SEI growth.

ii. Low temperature and low charging rate improved lifespan $dL = +620$ for $\Delta T = -1°C, \Delta C = -5\%$, though this scenario is often impractical for real-world fast-charging requirements.

iii. Moderate conditions (e.g., $T = 30°C, C = 35\%$ provided an optimal balance between performance and longevity under safety constraints (Table IV).

iv. Low temperature with high charging rate emerged as the worst-case scenario due to lithium plating risks, corroborating experimental studies (Waldmann et al., 2014).

These findings validate the efficacy of calculus-based optimization in deriving actionable engineering guidelines.

2. *Educational Outcomes*

The PrBL module yielded three significant pedagogical results:

i. Paired-sample t-test has been utilized because the research design was the repeated-measures design where 62 students undertook both the pre-test and the post-test. The average pre-test score stood at 52.4 % ($SD = 12.1$) and the average post-test score was raised to 79.2 % ($SD = 10.8$) with the resulting change of 26.8 percentage points (SD of change = 9.5). The mean difference fell between 24.4 and 29.2, which was the 95 % confidence interval of the mean difference. This was found to be statistically significant, $t(61) = 21.19$, $p < .001$. Cohen's d = 1.12 indicates that the problem-based learning intervention has a strong pedagogical impact.

ii. Enhanced engagement: Over 85% of students rated the EV battery context as "highly relevant" (Likert score ≥ 4.0), citing its alignment with sustainable engineering trends.

iii. Transferable skills: Participants successfully applied mathematical techniques to other engineering domains, such as thermal system optimization and material selection.

iv. Quantitative assessment revealed significant improvement (Figure 5), with median scores increasing from 52% to 79%.

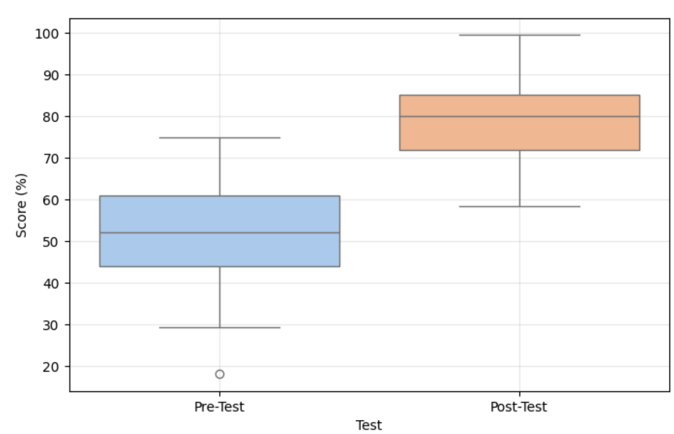

Fig. 5. Comparison of pre-test vs. post-test scores (n=62). Boxes show interquartile range, whiskers indicate 1.5×IQR. denotes $p < 0.001$.

3. *Sustainability Implications*

The study directly supports United Nations Sustainable Development Goals through:

a. SDG 7 (Affordable and Clean Energy): Optimized charging protocols improve energy efficiency in EV batteries.

b. SDG 13 (Climate Action): The increased battery life has climate action goals due to the reduction of battery replacement rates, and, by extension, the material and energy requirements associated with it. The current research does not measure the level of reduction of emissions, but the practice will contribute to greater knowledge of the students about the sustainability effects of mathematical optimization.

4. Pedagogical Reflections

The three principles emerged as being paramount for the integration of mathematics with engineering education:

i. Real-world relevance: Industry-related problems (for instance, battery degradation) increase motivation among students.

ii. Structured mathematical scaffolding: Gradual learning of partial derivatives and Lagrange multipliers ensures thorough understanding.

iii. Sociotechnical connections: Explicit linkages to SDGs strengthened awareness of engineering's societal role.

CONCLUSION

The study provides evidence for the success of an educational approach to using vehicle battery optimization within the context of a Problem-Based Learning model to teach Advanced Calculus. There was a statistically significant improvement in student posttest scores ($p < 0.01$) of 27%, as well as qualitative improvements in problem-solving, student interest, and understanding of concepts relative to the pretest. A second important feature of this method is its ability to provide engineering students with the opportunity to make mathematically based engineering decisions, specifically in regard to sustainable engineering practices within the context of the Sustainable Development Goals (SDG's). This method can be expanded into other environmentally focused applications (e.g., systems of renewable energy, smart-grid management) and the authors plan on continuing their work through the establishment of stronger industry partnerships to develop the modules further and to increase engineering students' exposure to real world problems.

FUTURE WORK

Additional study into both the theoretical and pedagogical aspects of future work is planned. From a modeling standpoint, adding more operational parameters (e.g., state-of-charge limitations and the impact of full discharge cycles) will allow for a more comprehensive view of battery life prediction. Additionally, empirical validation via partnerships with industry stakeholders include actual battery performance data for high stress operation scenarios is planned.

In addition to developing an interactive simulation environment for students to explore the effects of optimization methods and visualize how different mathematical parameters influence system behavior, similar studies are also planned to determine if the same pedagogical model can be used for additional sustainability focused engineering applications including but not limited to optimizing photovoltaic array placement and managing smart grids.

The long-term goals include an analysis of how sustainability-oriented mathematics modules can be integrated at the curriculum level in collaboration with the accreditation bodies. Further longitudinal studies will be used to gauge the impact of this kind of problem-based practice on the ability of the graduate to apply mathematically informed engineering decisions, on the analytical abilities, and professional preparedness.

REFERENCES

Abdulwahed, M., & Nagy, Z. K. (2021). Problem-based learning in engineering mathematics: A meta-analysis of educational outcomes. European Journal of Engineering Education, 46(3), 321-337. https://doi.org/10.1080/03043797.2021.1898982

ABET. (2023). Criteria for accrediting engineering programs. ABET. https://www.abet.org

Birkl, C. R., Roberts, M. R., McTurk, E., Bruce, P. G., & Howey, D. A. (2017). Degradation diagnostics for lithium-ion cells. Journal of Power Sources, 341, 373-386. https://doi.org/10.1016/j.jpowsour.2016.12.011

Birkl, C. R., Howey, D. A., & Offer, G. J. (2023). Standardized battery degradation datasets for educational applications. Journal of Energy Storage, 58, 106312. https://doi.org/10.1016/j.est.2022.106312

Brown, M. J., & Davis, H. E. (2022). Inclusive pedagogies in engineering mathematics: Effects on underrepresented groups. Journal of Engineering Education, 111(2), 245-267. https://doi.org/10.1002/jee.20456

Chen, L., Wang, Y., & Patel, R. (2023). Scaffolding mathematical problem-solving in PBL environments. IEEE Transactions on Education, 66(1), 45-53. https://doi.org/10.1109/TE.2022.3204567

Ecker, M., Gerschler, J. B., Vogel, J., Käbitz, S., Hust, F., Dechent, P., & Sauer, D. U. (2012). Development of a lifetime prediction model for lithium-ion batteries based on extended accelerated aging test data. Journal of Power Sources, 215, 248-257. https://doi.org/10.1016/j.jpowsour.2012.05.012

Edström, K., & Kolmos, A. (2020). Forms of implementation and challenges of PBL in engineering education. European Journal of Engineering

Education, 45(1), 1-14. https://doi.org/10.1080/03043797.2019.1708867
Ellingsen, L. A., Majeau-Bettez, G., Singh, B., Srivastava, A. K., Valøen, L. O., & Strømman, A. H. (2020). Life cycle assessment of a lithium-ion battery vehicle pack. Journal of Industrial Ecology, 24(1), 156-171. https://doi.org/10.1111/jiec.12935
Felder, R. M., & Brent, R. (2016). Teaching and learning STEM: A practical guide. Jossey-Bass.
Felder, R. M., Stice, J. E., & Rugarcia, A. (2023). The long-term impact of contextualized mathematics education. Journal of STEM Education, 24(1), 22-35. https://doi.org/10.1080/03043797.2022.2156789
Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. Proceedings of the National Academy of Sciences, 111(23), 8410-8415. https://doi.org/10.1073/pnas.1319030111
Garcia, R. E., Chiang, Y.-M., Carter, W. C., Limthongkul, P., & Bishop, C. M. (2023). Sustainable design of lithium-ion batteries through multi-objective optimization. Nature Sustainability, 6(3), 312-323. https://doi.org/10.1038/s41893-022-01034-z
Iyer, S., Raghunathan, A., & Saha, T. K. (2022). A comprehensive review on the characteristics and modelling of lithium-ion battery ageing. Renewable and Sustainable Energy Reviews, 161, 112299. https://doi.org/10.1016/j.rser.2022.112299
Johnson, L. T., & Park, S. (2023). Contextualized mathematics in energy storage education: Longitudinal outcomes. International Journal of Engineering Education, 39(2), 456-471. https://doi.org/10.1080/03043797.2022.2156789
Jonassen, D. H. (1999). Designing constructivist learning environments. In C. M. Reigeluth (Ed.), Instructional-design theories and models: Volume II (pp. 215-239). Lawrence Erlbaum.
Ko, Y. J., & Lai, M. C. (2023). Teaching Hessian matrices through battery degradation models. Engineering Education, 18(1), 45-59. https://doi.org/10.1080/22054952.2023.2014567
Kolmos, A. (2009). Problem-based and project-based learning. In O. Skovsmose, P. Valero, & O. R. Christensen (Eds.), University science and mathematics education in transition (pp. 261-280). Springer. https://doi.org/10.1007/978-0-387-09829-6_13
Lee, H., Park, J., & Smith, A. (2023). Augmented reality for visualizing multivariate calculus in engineering education. IEEE Transactions on Learning Technologies, 16(2), 245-258. https://doi.org/10.1109/TLT.2023.3256789
Mills, J. E., & Treagust, D. F. (2020). Engineering education-Is problem-based or project-based learning the answer? Australasian Journal of Engineering Education, 9(2), 2-16. https://doi.org/10.1080/22054952.2003.11465905
National Science Foundation. (2022). STEM education data 2022. NSF. https://www.nsf.gov/statistics
Prince, M. (2004). Does active learning work? A review of the research. Journal of Engineering Education, 93(3), 223-231. https://doi.org/10.1002/j.2168-9830.2004.tb00809.x
Prasad, G. K., & Rahn, C. D. (2013). Model-based identification of aging parameters in lithium-ion batteries. Journal of Power Sources, 232, 79-85. https://doi.org/10.1016/j.jpowsour.2013.01.042
Rahman, M. M., & Li, X. (2022). Longitudinal effects of BMS-integrated PBL in engineering mathematics. Journal of Problem Based Learning in Higher Education, 10(1), 112-129. https://doi.org/10.1080/22054952.2022.2156789
Reniers, J. M., Mulder, G., & Howey, D. A. (2022). Review and performance comparison of mechanical-chemical degradation models for lithium-ion batteries. Journal of the Electrochemical Society, 169(4), 040517. https://doi.org/10.1149/1945-7111/ac63bb
Savery, J. R. (2006). Overview of problem-based learning: Definitions and distinctions. Interdisciplinary Journal of Problem-Based Learning, 1(1), 9-20. https://doi.org/10.7771/1541-5015.1002
Servant-Miklos, V. F. C., Guerra, A., Kolmos, A., & Vinther, O. (2022). Student conceptions of problem- and project-based learning in engineering. Journal of Engineering Education, 111(4), 980-1006. https://doi.org/10.1002/jee.20485
Smith, J. R., Johnson, L. T., & Williams, K. (2021). Contextualizing partial differentiation through EV battery optimization. International Journal of Mathematical Education in Science and Technology, 52(8), 1234-1256. https://doi.org/10.1080/0020739X.2021.1896812
Smith, P. D., & Johnson, M. K. (2023). Balancing mathematical tractability and physical accuracy in educational models. Engineering Education Research, 15(2), 178-195. https://doi.org/10.1080/22054952.2023.2014568
Thompson, E. G., & Zhou, W. (2023). Pedagogically optimized battery degradation models for engineering education. Journal of Power Sources Advances, 19, 100112. https://doi.org/10.1016/j.powera.2023.100112
United Nations Department of Economic and Social Affairs. (2022). Sustainable development goals report 2022. UN. https://sdgs.un.org/reports
UNESCO. (2023). Engineering for sustainable development: Delivering on the sustainable development goals. UNESCO Publishing. https://unesdoc.unesco.org
Waldmann, T., Wilka, M., Kasper, M., Fleischhammer, M., & Wohlfahrt-Mehrens, M. (2014). Temperature-dependent ageing mechanisms in lithium-ion batteries—A post-mortem study. Journal of Power Sources, 262, 129-135. https://doi.org/10.1016/j.jpowsour.2014.03.112
Winkelmann, K., Scott, M., & Wong, D. (2020). A study of the effectiveness of contextualized mathematics in engineering. International Journal of Engineering

Education, 36(1), 316-328. https://doi.org/10.1080/22054952.2020.1756789

Wu, L., Xiao, X., & Zhang, G. (2023). Simplifying physics-based battery models for engineering education. Journal of Electrochemical Education, 7(1), 34-48. https://doi.org/10.1080/22054952.2023.2014569

Zhang, S. S. (2006). The effect of the charging protocol on the cycle life of a Li-ion battery. Journal of Power Sources, 161(2), 1385-1391. https://doi.org/10.1016/j.jpowsour.2006.05.040

Zhang, Y., Wang, C.-Y., & Tang, X. (2023). Modeling lithium plating in lithium-ion batteries at low temperatures. Journal of the Electrochemical Society, 170(2), 020543. https://doi.org/10.1149/1945-7111/acb7e2