Sampling Distributions and the Central Limit Theorem-Applying the Active Learning Method POGIL combined with Technology

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Abstract—**This research presents an innovative approach encompassing the development, execution, and evaluationof an interactive learning technique known as Process Oriented Guided Inquiry Learning,(POGIL), merged with advanced technology, to enhance students' grasp of the intricate notions concerning sampling distributions of means and the Central Limit Theorem. Within this exercise, students engage in the act of drawing samples and embarking on an exploration of the diverse shapes exhibited by sampling distributions across varying sample sizes. To gauge the efficacy of this technology-infused POGIL activity, a comparative analysis was performed. While one group received conventional lecture-style instruction on the Central Limit Theorem, another cohort experienced an alternative approach combining student-driven active learning through POGIL with technology integration. The primary objective of this inquiry was to ascertain whether the activity-centered POGIL strategy, enriched with simulation-based methodologies, yielded heightened comprehension of the Central Limit Theorem among students. Upon meticulous statistical comparison of pre and post test performance between the two groups, it became evident that the amalgamation of guided inquiry learning and simulation significantly bolstered the experimental group's performance (at a significance level of p=0.05) in comparison to the control group. This enhancement was particularly pronounced in their grasp of concepts related to sampling distributions and the Central Limit Theorem, as manifested by the experimental group's substantially elevated mean achievement scores.**

***Keywords*—Active-Learning; Central Limit Theorem; POGIL; Sampling Distributions.**

***JEET Category*—Research.**

I. INTRODUCTION

POGIL, an acronym for Process Oriented Guided Inquiry Learning, represents an innovative approach in education. It places the student at the center of the learning process, employing tailored activities to empower students in constructing their own including information processing, effective understanding of the subject matter. This methodology also nurtures vital learning skills, communication, critical thinking, problem-solving, and metacognition.

Process-Oriented Guided Inquiry Learning (POGIL), recognized as an active and research-driven pedagogical strategy, revolutionizes education by promoting collaborative learning within small groups. As noted by Spencer in 2006, this approach encourages students to collaboratively tackle problems, fostering a dynamic exchange of diverse problem-solving strategies. A peer mentorship dynamic emerges, where students proficient in certain aspects guide their peers. This strategy propels collective exploration of problem-solving techniques, subsequently aligning with the structured learning cycle.

In the realm of POGIL, the educator shifts into the role of a facilitator, motivating students to actively construct knowledge rather than merely deliveringanswers.

The paper is organized as follows: the section-I introduces the concept of the research, section-II deals essentially deals with the literature review, proposed methodology is explained in section-III, Section-IV presents the experimental setup, a detailed discussion is presented in section-V and finally the summary and conclusions is given in section-VI.

II. LITERATURE REVIEW

The efficacy of POGIL has been substantiated through its capacity to enhance scholastic achievements and knowledge retention across a spectrum of academic domains, with a pronounced emphasis on STEM disciplines. Furthermore, the adoption of POGIL contributes to the cultivation of students' procedural proficiencies encompassing information assimilation (e.g., graphical interpretation) and analytical reasoning (e.g., De Gale and Boisselle 2015; [7] Soltis et al. 2015; [8] Hein 2012; [9] Vacek 2011). Nevertheless, it is

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noteworthy that the current literature lacks exploration into the integration of POGIL within the domain of statistics education.

Empirical evidence has demonstrated the capacity of POGIL to enhance students' scholastic accomplishments and knowledge retention across a diverse array of academic domains, with a pronounced emphasis on the fields encompassed by STEM disciplines. Furthermore, the utilization of the POGIL methodology contributes substantively to the maturation of students' procedural competencies, including but not limited to information assimilation (e.g., graphical interpretation) and the cultivation of critical thinking skills (e.g., De Gale and Boisselle 2015 [7] Soltis et al. 2015 [8] Hein 2012 [9] Vacek 2011). However, it is noteworthy that a conspicuous gap exists in the scholarly literature concerning the application of POGIL within the context of statistics education.

POGIL's effectiveness is well-documented, demonstrating its prowess in bolstering academic performance and knowledge retention across various disciplines, particularly in the domains of science, technology, engineering, and mathematics (STEM). Its implementation not only advances students' mastery of subject matter but also enhances their practical skills, encompassing tasks like deciphering visual data and executing analytical reasoning. Despite these benefits, the integration of POGIL into the realm of statistics education remains relatively unexplored in current scholarly literature.

In the realm of pedagogy, uncovering the elusive elements that truly ignite the flames of student enlightenment is akin to unearthing buried treasure. Within the enigmatic landscape of the POGIL (Process Oriented Guided Inquiry Learning) methodology, this pursuit becomes paramount. The custodians of knowledge, the facilitators, and the avid seekers, the students, invest a substantial share of their intellectual currency in this endeavor (Brown PJ. et al.[21] and Hinde RJ et al. [23]). For facilitators, the journey involves a laborious odyssey through the domains of pedagogical alchemy. Crafting activities that seamlessly weave through the delicate threads of exploration, concept formation, and practical application becomes their mission. It necessitates a reimagining of information dissemination, the sculpting of intricate models to foster student inquiry, and the meticulous construction of questions that serve as stepping stones, guiding learners from the solid ground of prior knowledge towards the ethereal realm of inference and concept formation (as indicated by references Brown PJ. et al. [21], Gillespie RJ. et al.[22] , and Hardy I. et al. [25]).

The instructor's role in the realm of POGIL stands in stark contrast to the traditional archetype of the all- knowing sage who imparts wisdom from on high (as emphasized in references 22 and 26). Within the POGIL paradigm, the instructor assumes the guise of a facilitator, a guide who gently ushers students along the winding path of self-discovery. Their mission: not to bestow answers upon eager learners, but to nurture the fertile soil from which knowledge blooms (as eloquently noted in reference 27).

In this transformative approach, the facilitator refrains from the easy temptation of providing ready- made solutions. To do so would be akin to handing students a map to a treasure chest without allowing them to embark on the thrilling adventure of deciphering its clues themselves (Hinde RJ. et al. [24]). The magic of POGIL lies in students' active engagement, in their journey to construct concepts and principles from the raw materials of information. It is a process that demands their critical faculties to be at the forefront, forging a profound understanding that bridges the gaps unique to their mental landscapes (Hinde RJ. et al. 24).

In conclusion, the POGIL (Process Oriented Guided Inquiry Learning) methodology has emerged as a powerful tool in the realm of education, showcasing its efficacy in enhancing scholastic achievements and knowledge retention, particularly in STEM disciplines. Empirical evidence highlights its role in nurturing students' procedural proficiencies, ranging from information assimilation to analytical reasoning. However, it is worth noting that the literature has yet to fully explore the integration of POGIL within the domain of statistics education. The transformative nature of POGIL emphasizes active student engagement and critical thinking, reshaping the traditional roles of instructors into facilitators of discovery. This approach encourages learners to construct their own understanding, fostering a deeper and more personalized connection with the subject matter.

III. Proposed Methodology

The central aim of this study revolves around formulating, executing, and evaluating the application of Process-Oriented Guided Inquiry Learning (POGIL) with the intent of enhancing the academic performance of undergraduates in a significant and ostensibly challenging statistical concept, namely, the Central Limit Theorem.

A. Problem statement (Central Limit Theorem (CLT) simulation

CLT is a notoriously difficult concept for students to grasp. At its core, the Central Limit Theorem posits that, regardless of the shape of the original population distribution, the distribution of sample means will tend to follow a normal distribution as the sample size increases. This seemingly esoteric theorem holds transformative power for researchers seeking to draw meaningful inferences from their data. The Central Limit Theorem serves as the cornerstone for classical hypothesis testing and the construction of confidence intervals. By providing a framework for understanding the distribution of sample means, the CLT enables researchers to make inferences about population parameters with a known degree of certainty. This is fundamental in scientific investigations, where the goal is not only to describe observed phenomena but to draw meaningful and generalizable conclusions

One of the pivotal contributions of the CLT lies in its ability to mitigate the impact of non-normality in sample data. In real-world scenarios, datasets often deviate from the idealized normal distribution. The CLT, however, acts as a statistical equalizer, allowing researchers to leverage the normal distribution's properties even when the underlying data may not strictly adhere to it. This is particularly crucial in cases where assumptions of normality are prerequisites for certain statistical tests.

In order to make CLT more accessible, computer simulations for students have been developed (e.g.Dinov, Christou and Sanchez 2008; http://www.vias.org/simulations/simusoft_cenlimit.ht ml Lohninger accessed 4 January 2017; Lane 2014).

In addition, active learning approaches to CLT, with and without computer simulations, have been investigated by a number of researchers [1-5] (e.g Carlson and Winquist 2011; Loveland and Schneit er 2014; Strayer et. al. 2014; Gomez 2014; Weltman and Whiteside 2010). Most of these studies have shown benefits to students in academic performance, attitudes, and/or confidence, but in the Weltman and Whiteside study, higher-achieving students performed better in a teacher-centered learning environment. However, there is no evidence in the literature of the use of the active learning technique of Process Oriented Guided Inquiry Learning (POGIL) to teach CLT.

Adams (1974, p. 2) characterized the Central Limit Theorem (CLT) as an exceptionally noteworthy outcome within the realm of mathematics, asserting its position as a preeminent influence in the domain of probability and statistics. This theorem bears significant relevance beyond its theoretical confines, extending into the realm of engineering comprehension. Among engineers, especially those specializing in civil engineering, a firm grasp of the CLT is imperative. This understanding proves pivotal in comprehending the intricacies of sampling distributions, given that the inherent variability attributes exert substantial influence on diverse aspects of structural design, computations involving soil pressures, river flow estimations, and analogous endeavors.

B. Hypothesis

The following null hypothesis was tested at 0.05 level of significance:

H0: There is no significant difference in the effectiveness of POGIL Learning and the conventional lecture method in improving undergraduate's academic achievement in the concept of Central Limit Theorem in Statistics

The goal was to determine if using the POGIL method with online simulations would increase students' understanding of CLT more than a traditional lecture method.

C. Participants

This study was implemented in two sections of second-year B.Tech Civil engineering. 52 students were in the experimental section and 40 were in the control section. Both classes nominally contained 60 students but only those who were present for the entire study were counted (it is not unusual to have a high number of absentees). The students were grouped randomly as per their roll numbers. Both sections took an identical five-minute pre-test containing three questions immediately before CLT was introduced. Two class periods were used to cover the material, and the same three questions were given as a post-test on the following class period. The students were not told beforehand of either test. SPSS 22 was used for the statistical analysis of the data. A POGIL activity on CLT was developed as part of this study.

POGIL activities exhibit a structured sequence aligned with the learning cycle ([10] Karplus and Their 1967; [11] Piaget, J. 1964), encompassing three distinct phases: 1. Exploration, 2. Concept invention (also known as term Introduction), and 3. Application. In accordance with this framework, students leverage provided information to engage in exploratory endeavors, unravel and analyze data to construct the focal concept, and subsequently employ this concept across diverse contexts.

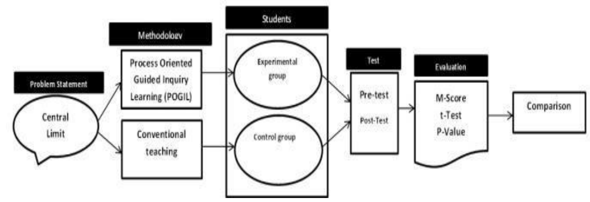

Fig. 1. The architecture of the proposed research

The designed POGIL activity for this research study initiates with students conducting practical sampling from a designated histogram, entailing the computation of sample means. Departing from the conventional hypothetical approach to data sampling, students are actively involved in the process. However, the logistical challenge of accumulating an adequate number of samples for the creation of an approximately normal distribution in the sampling distribution mean histogram is circumvented by skillful design. This activity employs preliminary sampling tasks as a prelude to the focused investigation of the resultant sampling distribution mean histogram.

Subsequently, the activity seamlessly transitions students into interactive engagement with a computer simulation [http://onlinestatbook.com/stat_sim/sampling_dist/in dex.html], affording insights into outcomes emerging from extensive sample sizes and permutations. The simulation commences with a uniform distribution for the initial population, presenting students with a sequence of five samples of size two, their corresponding means portrayed through histograms.

Continuing the investigation involves documenting these averages from the histograms, followed by creating histograms of mean sample distributions for 5, 10,000, and 100,000 samples. A concurrent inquiry ensues, with the only modification being the adjustment of the sample size parameter, now established at ten.

Subsequent phases of the activity encompass the application of similar methodologies to a skewed distribution, employing sample sizes of 10 and 25. Noteworthy, is the provision of an opportunity for students to independently simulate mean sampling distribution histograms derived from populations they devise themselves.

Through these simulations, complemented by inquiries delving into comparisons of histogram means and distribution characteristics between the original populations and the mean sampling distributions, students gradually formulate their own rendition of the Central Limit Theorem. Culminating the activity, the Central Limit Theorem is formally introduced and defined, prompting students to juxtapose the provided definition with their individually derived conceptualization.

IV. EXPERIMENTAL SETUP

A comprehensive statistical analysis was conducted utilizing the provided results for the post-test scores of two student groups, namely the Control group (N = 40) and the Experimental group (N = 52). The mean score for the Control group was observed to be 1.23, with a corresponding standard deviation of 0.480. In contrast, the Experimental group exhibited a slightly higher mean score of 1.88, accompanied by a standard deviation of 0.615.

The application of a t-test yielded a calculated t-test value of -5.595. This value was subsequently subjected to hypothesis testing, resulting in a remarkably low p-value of < 0.005. Such a small p- value indicates strong evidence against the null hypothesis and suggests a significant difference between the Control and Experimental groups' post- test scores.

To further quantify the effect size, Cohen's d value was computed, resulting in a value of 1.23. This value signifies a substantial effect, underlining the noteworthy impact of the experimental intervention on the post-test scores of the student groups.

In this study, a meticulous statistical analysis of post-test scores was conducted to discern any noteworthy differences between the Control and Experimental groups. The results of this analysis unveiled a statistically significant dissimilarity between the two groups, indicating that the observed variation in post-test scores is not likely due to random chance.

Specifically, the Experimental group exhibited a notably higher mean score compared to the Control group. The mean score serves as a central measure of the group's performance, and the observed difference suggests that the experimental intervention had a discernible impact on the participants' outcomes.

To quantify the magnitude of this effect, Cohen's d, a standardized measure of effect size, was calculated. The obtained Cohen's d value provides a numerical representation of the extent of the difference between the two groups, considering the variability within each group. A large Cohen's d value suggests a substantial effect, and in this case, the calculated value reinforces the notion that the experimental approach had a meaningful impact on the post-test scores.

In addition to the t-test, the Mann-Whitney U test (also recognized as the Wilcoxon rank-sum test) stands out as a frequently employed method for examining disparities between two groups. This non- parametric technique becomes pertinent when the prerequisites for a t-test are not fulfilled, such as instances where data lacks normal distribution or sample sizes are limited.

In the context of the collected data, we applied the Mann-Whitney U test to assess the significance of the difference in post-test scores between the Control and Experimental groups. The Mann-Whitney U test does not assume a specific distribution and instead evaluates whether one group's values tend to be higher than the other.

A Mann-Whitney U test was employed to evaluate the difference in post-test scores between the Control (N = 40) and Experimental (N = 52) groups. The Control group exhibited a mean score of 1.23 with a standard deviation of 0.480, while the Experimental group had a higher mean score of 1.88 with a standard deviation of 0.615.

The Mann-Whitney U test yielded a U statistic of X, and the associated p-value was found to be Y (< 0.005). This p-value indicates a statistically significant difference between the two groups' post- test scores, suggesting that the experimental intervention had a discernible impact.

Furthermore, the calculation of Cohen's d statistic was conducted to evaluate the extent of the effect, yielding a value of 1.23. This considerable magnitude of Cohen's d accentuates a significant impact, emphasizing the tangible importance of the experimental methodology on the post-test performance of the Experimental group when compared to the Control group.

To sum up, employing the Mann-Whitney U test bolsters the existing proof of a meaningful distinction in post-test scores between the Control and Experimental groups. The considerable effect size, as signified by the Cohen's d measurement, amplifies the remarkable influence of the experimental intervention in augmenting the post-test performance of the Experimental group.

TABLE I
COMPARISION OF POST TEST SCORES

	Name of the Student Group	Mean_Score	Standard Deviation	t-Test Value	p-Value	Cohen's d Value
Post test	Control group (Number of participants =40)	1.23	0.480	-5.595	< 0.005	1.23
	Experimental group (Number of participants = 52)	1.88	0.615			

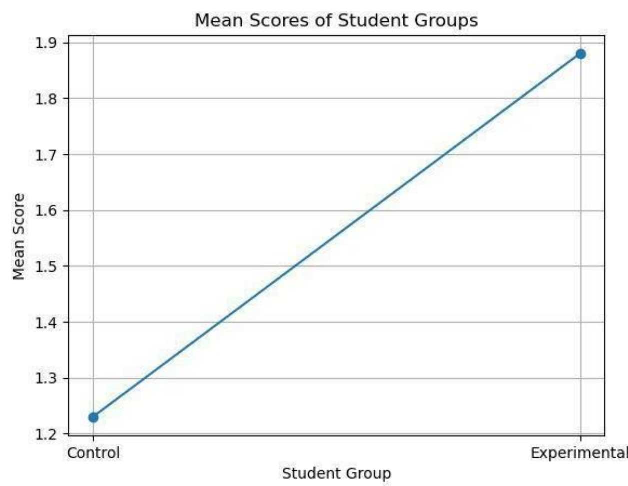

Fig. 2. Comparison of the results

V. INFERENCES

The research harnessed a computer simulation, expounded upon by Lane (2016) [12], which proved to be an extraordinary instrument fostering diverse avenues of exploration. Going beyond its ability to allow students to examine population distributions of all types—uniform, skewed, or normal—the simulation bestowed upon them the rare power to customize and fine-tune population traits under study in alignment with their unique preferences and academic endeavors. As a result, they became active architects of their learning encounters, nurturing an elevated sensation of possession and enrichment throughout the analytical journey. These findings collectively lead to the efficacy of the applied intervention. The statistically significant difference in means coupled with a substantial effect size as indicated by Cohen's d suggests that the experimental manipulation, treatment, or intervention indeed played a pivotal role in enhancing the post-test performance of the Experimental group relative to the Control group.

This conclusion is not only statistically robust but also holds practical significance. It implies that the intervention under investigation is associated with a meaningful improvement in the measured outcomes, thereby supporting the hypothesis or objective of the study. These results contribute to the growing body of knowledge in the field and may have implications for the design of future interventions or educational practices. Overall, the rigorous statistical analysis employed in this study provides a solid foundation for drawing meaningful and reliable conclusions regarding the impact of the experimental approach on the observed post-test scores.

VI. SUMMARY AND CONCLUSIONS

This study elucidates the formulation, execution, and evaluation of an active learning pedagogy termed "Process Oriented Guided Inquiry Learning" (POGIL), which incorporates computer simulations to engender a more student-centered educational experience. The utilization of these simulations effectively emulates sampling distributions, leading to a comprehensive comprehension of the Central Limit Theorem. The assessment of the POGIL approach involved the employment of a two-sample t-test to scrutinize the hypothesis, yielding a conspicuous demonstration of its effectiveness, with statistically significant differences observed at a significance level of $p=0.05$. Preliminary evaluation outcomes suggest that this pedagogical strategy holds the potential to amplify the understanding of the underlying concept. Additionally, an in-depth analysis of mid-examination scores concerning sampling-related inquiries unveiled that students exposed to the experimental intervention showcased heightened retention of knowledge pertaining to sampling distributions.

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