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Elegant Computational Frameworks For the Analysis of Cantilevers and Beams

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Abstract : In various fields like automobiles, construction, etc., the structural analysis of each component or sub-system must be done to ensure its safe operation. The structural analysis of these components entails the determination of parameters like shear force, bending moment at different locations. Usually, such computations are cumbersome, and hence a simplified approach is adopted, that involves drawing shear force diagram (SFD) and bending moment diagram (BMD) for the components. These diagrams can be effectively utilized to determine the dimensions of the components, select the appropriate material for the structure etc. Also, by utilizing the values of maximum shear force and bending moments, the maximum deflection in a beam or other structure can be ascertained. However, the process of drawing these diagrams is cumbrous and involves a lot of meticulous effort and time, which sometime poses a challenge in the effective teaching and learning of these concepts. The current paper reports the development of computational tools using the excel VBA platform and its implementation in the pedagogy of an undergraduate solid mechanics classroom. The developed tools can be easily employed to instantaneously draw the SFD and BMD diagrams for the beam under a variety of loading conditions, facilitating the inference-based learning of cantilevers and beams. Two distinct tools were developed, one for drawing the SFD and BMD of both cantilever and simply supported beams, and another one to determine the deflection and slope in the same two beams. The tools reported in the current manuscript can be effectively utilized for teaching by the demonstration of parametric variations under various loading conditions, for improved comprehension of the concepts and self-learning as well as in real world engineering to get preliminary design guidelines. Upon the development of these computational tools, these have been introduced to undergraduate mechanical engineering class of a sizable population and student responses regarding the efficacy of such tools in aiding the learning process has been recorded through an anonymous feedback. The subsequent hypothesis testing and obtained p-values strongly justify the extreme usefulness of the tools, both as a teaching and learning strategy.

Keywords : SFD, BMD, cantilever beam, simply supported beam, computational tool, Excel VBA

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1. Introduction

Safety isn't expensive, it's priceless. Safe operation of engineering components is crucial to the industry and many techniques are thus employed to ensure the safe operation of structures. Static structural analysis is one of the important tools to ascertain the safe operation of such component (Mankar et al., 2015). In fact, it is routinely applied in industry to find out the dimensions, suitable materials, and other parameters for the design of different engineering components. Static structural analysis entails the computation of maximum shear stress and bending moments of components under different load conditions. This is further achieved by drawing Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) for the components, and yield shear force and bending moment, respectively at different locations. These analytical tools used in combination with the structural analysis can aid in structural design for elements such as a beam. These diagrams can provide for the size, type and materials of a structural member such that a given set of loads can be safely supported (Vashist, 2013). In addition, these also provide for the deflection of a beam by techniques such as the moment area method. However, in spite of its immense engineering utility, due to lengthy and time-consuming calculations, exercises in the classroom have been limited and the underlying complex procedures make it difficult to demonstrate the parametric variations for teaching and learning purposes (Engeda, 2010). As a result, many students find these techniques difficult and cumbersome, something that impedes the learning process. But, the spreadsheets have opened up a simple and powerful way of performing design calculations, that relieves students from time-consuming chores and hence complex engineering procedures could be carried out and demonstrated with ease (Engeda, 2010; Dwivedi et al. 2022a; Agarwal et al. 2022).

The following factors make it difficult for undergraduate and graduate students to comprehend the concepts of shear force and bending moment and apply these concepts to industrial and practical applications: high-level mathematical calculations including partial differential equations, usage of vectors and tensor calculus etc. In a majority of cases, the solutions to these are obtained through intricate experiments and perplexing computations (Dwivedi et al. 2022b). In addition, there is an additional drawback of the conventional calculation methods – at any given time, it yields the results for only one fixed loading condition. In order to analyze the situation at a different amount of load, the calculations need to be done afresh for every single value of given load. This makes it difficult to study and compare distributions over the entire beam for these different cases. These problems are further compounded when the loading condition at one of the end needs to be changed. Clearly, the extra insight that one may easily gain by such comparative parametric study is missed out owing to the absence of iterative computer based tools, and the chances of error in such repetitive calculations are indeed high. Some interactive and computer based tools need to be developed which help students to learn such concepts with their own pace (Lumsdaine & Ratchukool, 2003). The current paper includes a description of two such tools that have designed for a better understanding of the subject. The tools are very user-friendly and very easy to use for everyone.

These methods of drawing SFD and BMD help in finding the shear force and bending moment for the beam at a different location for the given loading condition (Singh, 2021). These are generally the plot between shear force / bending moment vs. distance from the fixed point. As described before that after the calculation of share force and bending moment the value of deflection and slope can also be calculated for better visualization. Generally, the value of maximum deflection is found out to account for the worst possible behavior of the beam for different loading conditions (Egelhoff & Odom, 2014). Indeed, many industry use commercially available software for running the static structural analysis. However, to fully comprehend the working of these software, a deeper understanding of parameters like SFD, BMD, deflection, etc. is a must and a good foundation and depth in the subject knowledge is also required. Unfortunately, many undergraduate students give up on this learning curve as the cumbersome process of design calculations divests them of the required interest in the subject (Sadid & Wabrek, 2009). This paper contains the description of some great and interesting tools which help students to better understand the subject and also help them to visualize the behavior of the plots with different parameters (Hossain & Al-Faruk, 2017). Two distinct computational tools have been designed using the Excel-VBA platform. The first tool pertains to the SFD and BMD of two types of beams - cantilever beam and simply supported beam. The second tool provides the deflection and slope for the same cantilever and simply supported the beam at a different location. It is expected that these tools will help the students learn about these complex engineering topics (Jong et al., 2006).

2. Background

The process of drawing SFD and BMD for a cantilever beam consists of many steps and after getting the value of maximum stress and moments, one can calculate the deflection and slope of the beam (Singh, 2021). The parameters for the calculation can be understood by the following diagram, where A = fixed point

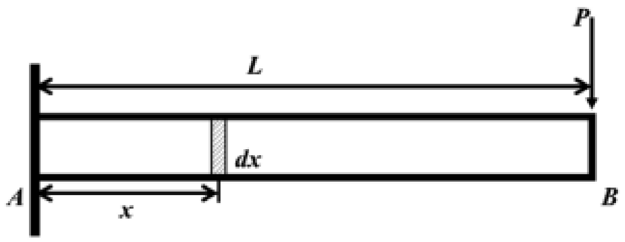


Fig. 1: Cantilever Beam With Applied Load At the Free End.

x = distance of the point from fixed point

dx = thickness of the small element under consideration

B = point at free end

P = load at free end

For drawing the SFD and BMD, first one needs to balance the force and moment in the system (Beléndez et al., 2005). For the cantilever beam, the balancing loads are at the fixed point. In this case, there is a singular moment and a vertical force on the fixed point which balance all the applied load on the beam. After balancing these loads, the beam is to be divided into sections. The number of the sections depends on the number of loads and moments applied on the beam (Mischke, 1978). For "n" number of loads, there will be "n+1" sections and each section is selected in such a way that there is no other section between the two loads (Nishawala, 2011). Now after the beam has been divided into sections, the calculation for each section has to be done separately. This is done by assuming a very small element of thickness dx at the distance x from the fixed point. At this point x, one can balance all the force at the left side of the beam with load and moment (Den Hartog, 1987) and the value of that load and moment at x gives us the shear load and bending moment at that point. Clearly, in these expressions, as the force and moments are substituted in the terms of x, the equation of shear force and bending moment are with respect to x as well. Hence, by plotting this equation for all the local sections, the SFD and BMD of the system can be obtained (Beer et al., 2006).

Let us assume the above-given system where a point load is acted at "*B*" distance from the fixed point. Now the balancing load at a fixed point is "*P*" and the consequent moment is "$P \times a$". Next, to do the calculations for the system, it has to be divided into two sections. The first section is *AX* and another is *XB*(Morley, 1920). For Section AX, Shear force = *P*, Bending moment = $P \times x$. Similarly, for Section *XB* can be Shear force = 0 and Bending moment = $P \times x$.Based on these equations, the SFD and BMD of the system can be easily drawn (Mohan, 2011).

Next, for the calculation of deflection and slope, first the value of shear force and bending moment have to be determined. After completing the process of depicting SFD and BMD, one can readily draw the slope and deflection curve for the same beam. The process for drawing this curve can easily be understood by the following method (Liu, 2017).

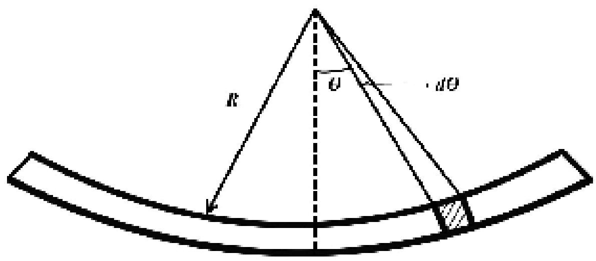


Fig. 2: Schematic showing the radius of the curvature of the beam upon application of a force.

Typically, the radius of curvature obtained by the beam during bending, upon application of a force can be represented by the following expression (Beer et al. 2006),

where,

R = radius of curvature
M = moment
E = modulus of elasticity, and
I = moment of inertia.

Further, the inverse of radius of curvature can also be expressed as :

$$\frac{1}{R} = \frac{d\Theta}{dx} = \frac{(d^2 y)}{(dx^2)}$$

So, after solving the above equations, the value of the slope as $\frac{dy}{dx}$ and value of deflection as y can be readily determined (Da Silva, 2005). In addition, some standard formulas for deflection and slope for some standard cases are as follows.

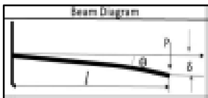
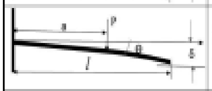
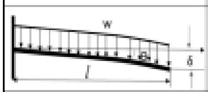
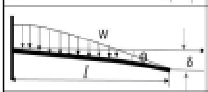
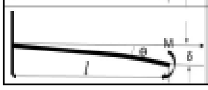
Beam Diagram	Beam Type	Slope(θ)	Deflection(δ)
	Cantilever beam With Load P at End Point.	$\frac{P l^2}{2 EI}$	$\frac{P x^2}{6EI}(3l - x)$
	Cantilever beam With load P at any Point.	$\frac{P a^2}{2 EI}$	$\frac{P x^2}{6EI}(3a - x)$ For 0<x<a $\frac{P a^2}{6EI}(3x - a)$ For a<x<l
	Cantilever beam With Uniform Distributed Load w.	$\frac{w l^3}{6 EI}$	$\frac{w x^2}{24 EI}(x^2 + 6l^2 - 4lx)$
	Cantilever beam With Uniform Varying Load W.	$\frac{W l^3}{24 EI}$	$\frac{W x^2}{120 lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$
	Cantilever beam With Moment M at End Point.	$\frac{M l}{EI}$	$\frac{M x^2}{2EI}$

Fig 3 : Expressions For Slope and Deflection For Various Cases of A Cantilever Beam.

Description of the developed computational tool

As mentioned in the previous section, two distinct computational solver tools have been developed using VBA/Excel to give a better understanding of SFD, BMD, slope, and deflection calculations. The first is SFD and BMD for cantilever beam solver which can be downloaded from https://www.drkarnteaching.com/strength-of-materials-tools by the engineering students, instructors, or professionals. As shown in figure 4 below, the user can select the type of load acting on the cantilever beam, vary the length of the beam, the value of point load acting, the value of the moment acting, and the value of the uniformly distributed load. The type of load acting on the beam can easily be selected from the drop-down menu which offers users four options: point load at the edge of the beam, uniformly distributed load, the moment at the edge of the beam, and uniform distributed load with the beam. In addition, slider bars are also provided alongside each input so the user may easily select and enter the right values in the input column.

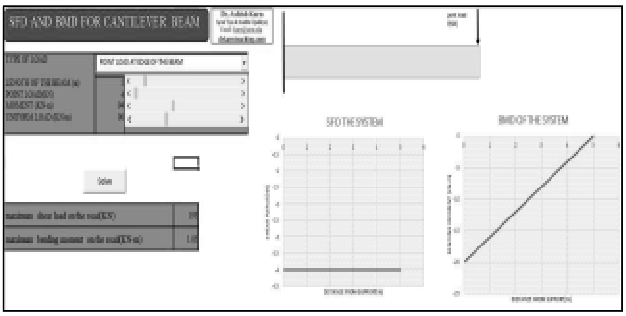


Fig. 4 : SFD and BMD Solver For A Cantilever Beam.

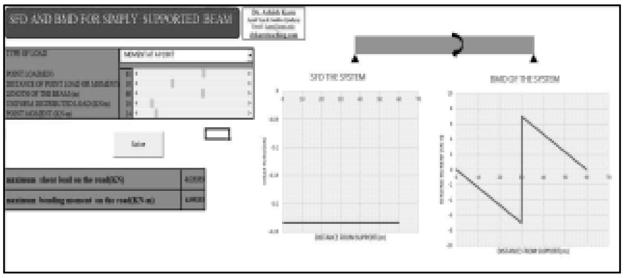


Fig 5.: SFD and BMD Solver For a Simply Supported Beam

Figure 5 shows the case of a simply supported beam having various inputs just like a cantilever beam. Upon feeding these desired inputs a solver button is also inserted in the spreadsheet that on clicking gives the result that is "Maximum shear load on the rod" and "Maximum bending moment on the rod". The tool does not merely compute the shear force and bending moment but also helps in visualizing the change in both parameters with respect to distance from a fixed support.

Next, figure 6 presents the slope and deflection for the cantilever beam solver. A spreadsheet is provided where the user can seek the various inputs like the type of load on the beam, length of the beam, point load, moment, uniform load value, modulus of elasticity, cross-section length, cross-section breadth, and uniform varying load. In addition, for the selection of the type of load drop-down boxes are presented which provide options to select various loading conditions

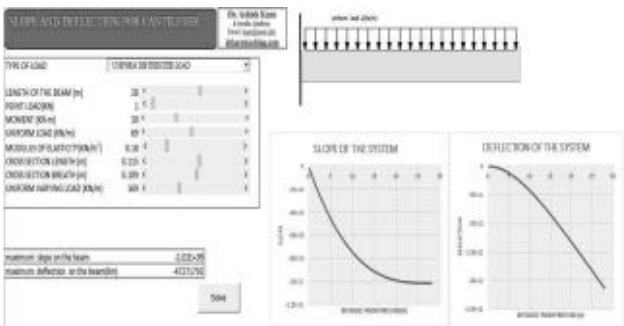


Fig. 6 : Slope and Deflection Solver for the Cantilever Beam

for the users. With drop-down boxes, slider bars are also provided alongside each input that helps the user to enter the right values in the input column for given parameters. The tool calculates the maximum slope on the beam and maximum deflection on the beam.

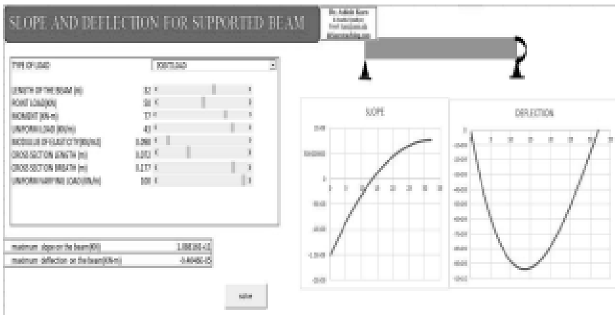


Fig. 7 : Slope And Deflection Solver for the Simply Supported Beam

Figure 7 shows the case of calculation of slope and deflection in a simply supported beam. The tool does not merely calculate the maximum slope on the beam and maximum deflection on the beam but also helps in visualizing the variations of slope on the beam and deflection on the beam with respect to the distance from the support end.

User Surveys and Hypothesis Testing

Finally, after completing the development of the computational tool now to test the efficacy of the same in enhancing the teaching and learning of the subject Mechanics of materials have been tested by taking sample surveys of a variety of users and the resulting data has been analysed using hypothesis testing principles, to arrive at some substantive conclusions. To collect the data, the developed tool was uploaded on the course website , and it was given access to the Mechanical Engineering, Mechatronics Engineering, and Automotive Design Engineering students who have studied the subject during their studies. The tool was also open for review from the other students as well as the faculty/ research scholars who wished to provide the data and feedback for the innovative tool .

The survey form contained ten questions, and the users were requested to offer appraisals to these queries on a five-point scale, where 1 denoted a strong disagreement while 5 represents a strong agreement. Moreover, qualitative feedback was also taken from individuals concerning the different aspects of the computational tools, and their overall utility and user-friendliness. The first query specifically discusses the overall worth of this proposed tool as an advanced pedagogical initiative in the teaching and learning process, while the last question quantifies how such a usage provides students with an edge over a pedagogy of the same course/concept without such an intervention. The middle eight questions are furthermore aggregated under two heads: the first relates specifically to the students' discernments with respect to supporting self-learning. The second set identifies with the teaching and learning perspective, such as enabling the students to see the role of the integration of computers in the learning of deeper insights of the concepts and also in supporting the smooth conduction of educational exercises such as tutorial sessions for the students. The student responses (both qualitative and quantitative) indicate the overwhelming acceptance of this computational intervention in enhancing the teaching-learning process of the engineering concepts (Agarwal et al., 2022).

A. Overall acceptance of the new method

Figure 8 displays a pie chart of the user perceptions regarding the overall acceptance of the developed computational tool in enhancing the teaching-learning experience in cantilevers and beams. While analysing the user inputs, the entries on a numeric scale of one to five have been interpreted as ‘Don’t like, ‘Somewhat like’, ‘Okay’, ‘Good work’, and ‘Appreciate greatly’, respectively.

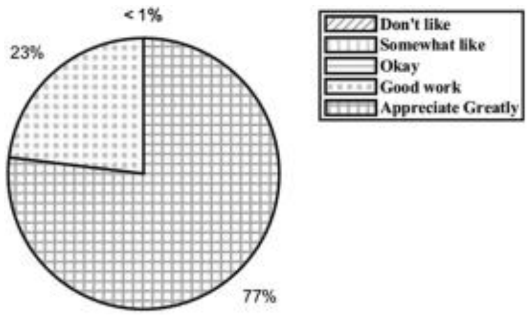


Fig. 8 : Students’ Perceptions Regarding the Role of Computational Tools Aiding Self-learning.

As per this adaptation, the figure shows that about 77% of the users provide a greatly positive endorsement for this tool, while another 23% of the population applaud this work. The rest of the population affirm the new computational tool to be ‘okay’ in its usefulness and none of the users provided a lesser rating for the developed innovative tool, which validates our claim regarding the overall usefulness of the tool.

B. User perceptions on aiding Self-learning

Figure 9 displays a bar plot of the students' perceptions regarding the role of the computational tool in aiding self-learning of the concepts related to shear force and bending moment diagrams for cantilevers. As the figure shows, under all the different segments such as the role of the tool in reducing the difficulty of the shear force concepts, promotion of cooperative learning, its usefulness in sustaining the interest of the students in the learning of mechanics and shear force and bending moment diagrams, as well as facilitating quality improvement of the course delivery, about 17 users have greatly appreciated this initiative, while another 9 users have applauded the initiative as a 'commendable work'. No user opted for the 'Don't like' option under any of the divisions under this group. This makes the cardinal role of these computational tools in assisting the self-learning of students, amply clear.

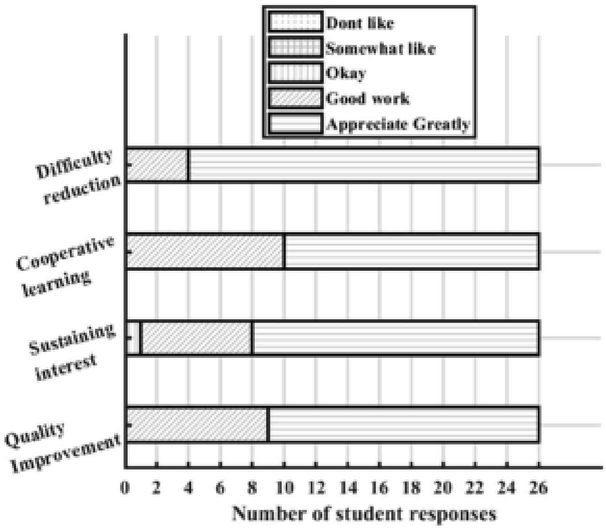


Fig. 9 : Students' Perceptions Regarding the Role of Computational Tools Aiding Self-learning.

C. User perceptions of Instructional aspects

The response of the users from another standpoint of improvement in the instructional aspects of the course delivery has then been examined. Figure 10 again shows that an enormous majority of the user population has commended the usage of the developed computational tools on shear force and bending moment diagram for cantilevers and beams by providing the highest and the next highest rating, respectively. There was hardly one user who 'somewhat liked' the usage of computational tools as far as the integration of computers in engineering professions is concerned. This is not surprising since the adoption of a completely novel pedagogy to engineering problem-solving through sophisticated tools may put some in a state of discomfiture, particularly if they are not comfortable with computers. This is expected since some students show a marked preference for the 'pen-and-paper' and 'pattern-recognition' approach to problem-solving. However, even this student seems to have appreciated the role of the proposed innovation in imparting real-life engineering skills, its importance in honing problem-solving skills during tutorial sessions, and that it can be befittingly used as an excellent lecture-demonstration to aid student learning. Overall, the data testifies to the effectiveness of the developed tools in enhancing instructional aspects of the course delivery.

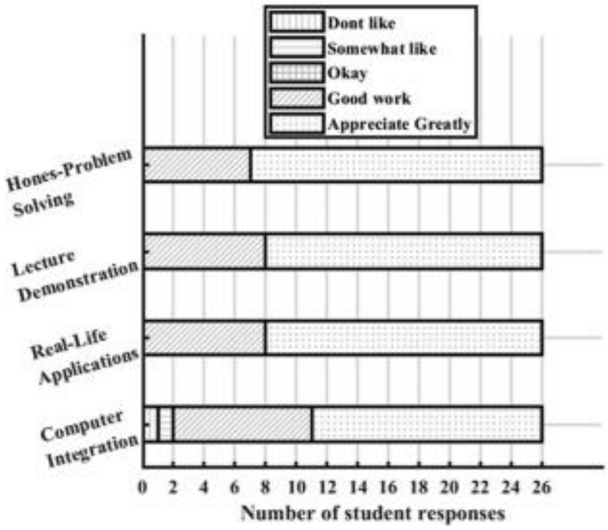


Fig. 10 Students' perceptions regarding the role of computational tools in promoting instructional aspects of the course.

D. Recommendation for Other Courses

Finally, it would be worth knowing the opinions of the users regarding the recommendation of such pedagogy to other engineering domains. This may be important to know before one can generalize the observations procured from a single mechanics course, and may extrapolate them to the other courses in mechanical engineering such as fluid mechanics or manufacturing technology. However, unlike other questions, while answering this question, the users were provided with only four options: 'Don't recommend', 'Just recommend', 'Moderately recommend', and 'Highly recommend. Figure 11 evinces the quantitative evidence of the user responses. This explicates the notion of the engineering users that such a pedagogy must be introduced in other engineering courses as well.

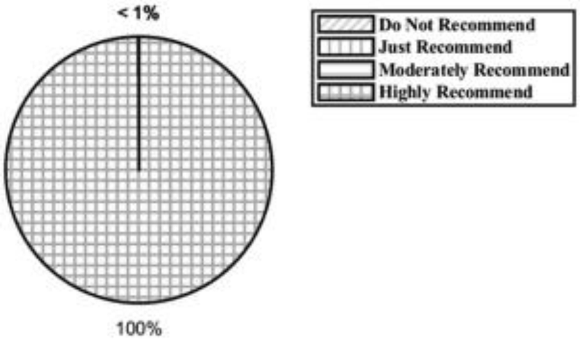


Fig. 11 : User Responses Regarding Their Recommendation Of Introduction of Such Tools in Other Engineering Courses.

E. Analysis of the Hypothesis testing results

Hypothesis testing is a way to find out whether a hypothesis concerning a population can be considered acceptable. A hypothesis is a presumption about something. The actual test begins by considering two hypotheses: the null hypothesis (H_o) and the alternative hypothesis (H_a). The alternative hypothesis is a claim about the population that is contradictory to H_o and what is naturally concluded upon rejection of H_o. Since the null and alternative hypotheses are contradictory, one must examine evidence to decide if there exists enough evidence to reject the null hypothesis or not. The evidence is in the form of sample data. After it has been determined which hypothesis the sample supports, two possible decisions could be made: "reject H_o" if the sample information favours the alternative hypothesis or "decline to reject H_o" if the sample information is insufficient to reject the null hypothesis. For instance, the first hypothesis (of the ten hypotheses presented in this manuscript) as presented in Table 1 can be written technically as follows:

H_0 The developed computational tool does not considerably improve the overall teaching -learning experience of the Subject.

H_a The developed computational tool considerably improves the overall teaching-learning experience of the Subject.

Based on population and type of data, many tests can be used for hypothesis testing. Out of all these several tests, the "t-test" is considered most suitable for the present population, and thus both 'One-Tailed test' and 'Two-tailed t-test' have been conducted to calculate the p-value, which in statistics is the probability of obtaining results at least as extreme as the observed results of a hypothesis test, assuming null hypothesis to be correct. Thus, a smaller p-value (< 0.05) implies that the alternate hypothesis is correct. The "t-test" feature is in-built in excel under the data analysis tool pack section in which variable range can be selected and it gives the output of mean, several observations, variance, and p-value for 'One-tailed t-test' and 'Two-tailed t-test'.

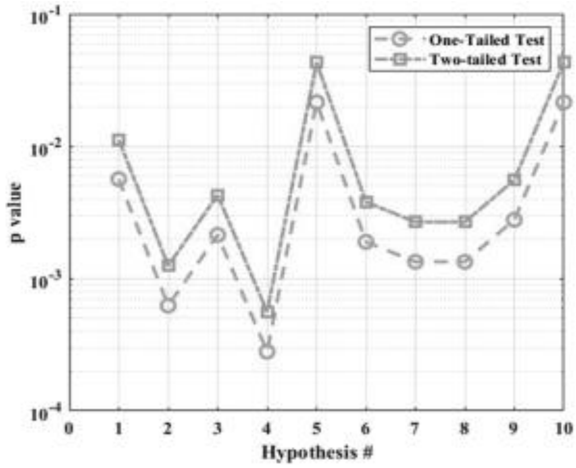


Fig. 12: P Values for All Hypotheses Presented in Table 1, Using 'one-tailed T-test' and 'two-tailed T-test'.

Figure 12 presents the p values for all the hypotheses. As the figure shows, there is a slight variation between the p values of different hypotheses, but the trends of variation of the One-Tailed test and Two-tailed test remain the same. Further, the upper bound of these p-values are 0.002 and 0.0044, and since the p-value is lesser than 0.05 for all the hypotheses, these represent statistically significant test results, i.e., the null hypothesis is false and must be rejected. Thus, the results of hypothesis testing clearly and substantiates the usage of these computational tools in the Strength of material education in particular, and to some extent, engineering education in general. It is expected that the development of such computational tools may open a new foray into which concerted development of engineering pedagogy could take place, and which has the potential of transforming engineering education in terms of delivery and self-learning.

Conclusions

Two powerful, versatile and educational program has been developed to generate graphs for visualization of SFD, BMD, slope, and deflections in a cantilever/ simply supported beam which, in turn, can be numerically simulated with adequate computer code. The approach developed in this paper for solving SFD, BMD, slope, and deflections problem provides a reliable, efficient means of explicitly determining a variety of design, operating, and calibration parameters for cantilever beams. Manual solutions to these types of problems require time and are a difficult process as it involves solving complex equations. This approach also provides an efficient technique to enhance real-time modelling, which requires the reliable, fast calculation of many of the parameters discussed in this paper. The paper demonstrates the use of spreadsheets as an effective educational tool for the calculations of SFD, BMD, Slope, and deflections related to beams. Certain features like IF statements, and FOR loop, inherent to spreadsheets were used to improve the efficiency of the solution, and the macros were used to automate the solution steps. The developed computational tools have been hosted on a self-created open-source website freely to be used by the students and engineering instructors alike. Not only does the current paper describe the development of such tools, but it has also provided the exact methodology, using which anyone can design such a tool for himself. Further, the present paper has also substantiated the efficacy of such computational tools in the engineering pedagogy, which ties in well with the new educational policy, NEP-2020 of Government of India that has put an emphasis of integration of computers into pedagogy for effective teaching and learning process. This paper can be categorized as an effort in this direction whereby numerical and graphical presentation and visualization of data has been employed by implementing spreadsheet programs along with VBA functions. It is hoped that the proposed intervention may prove to be beneficial aid for the students, instructors and the practicing professionals in the area of design of beams under different loading conditions.

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